

AD-A107 669

GEORGE WASHINGTON UNIV WASHINGTON DC PROGRAM IN LOGISTICS F/G 13/13
A CLOSED FORM LOCAL SOLUTION OF A NONLINEAR STRUCTURAL DESIGN P--ETC(U)
AUG 81 A V FIACCO, A GHAEMI
N00014-75-C-0729

NI

UNCLASSIFIED

SERIAL-T-449

1 OF
40 A
REED

END
DATE
FORMED
1-82
DTIC

4
ADA107669

LEVEL II

12

THE
GEORGE
WASHINGTON
UNIVERSITY

STUDENTS FACULTY STUDY R
ESEARCH DEVELOPMENT FUT
URE CAREER CREATIVITY CC
MMUNITY LEADERSHIP TECH
NOLOGY FRONTIERS DESIGN
ENGINEERING APPENDIX
GEORGE WASHINGTON UNIVERSITY

DTIC
SELECTED
NOV 20 1981
S D
D

81 11 10 961

SCHOOL OF ENGINEERING
AND APPLIED SCIENCE

THIS DOCUMENT HAS BEEN APPROVED FOR PUBLIC RELEASE AND SALE; ITS DISTRIBUTION IS UNLIMITED



FILE COPY

12

LEVEL

A CLOSED FORM LOCAL SOLUTION OF A
NONLINEAR STRUCTURAL DESIGN PROBLEM IN
TERMS OF THE DESIGN PARAMETERS

by

Anthony V. Fiacco
Abolfazl Ghaemi

Serial T-449
10 August 1981

The George Washington University
School of Engineering and Applied Science
Institute for Management Science and Engineering

Accession For	
NTIS GRA&I <input checked="" type="checkbox"/>	
DTIC TAB <input type="checkbox"/>	
Unannounced <input type="checkbox"/>	
Justification	
By _____	
Distribution/ _____	
Availability Codes	
Dist	Avail and/or Special
A	

Program in Logistics

Contract N00014-75-C-0729
Project NR 347 020
Office of Naval Research

This document has been approved for public
sale and release; its distribution is unlimited.

DTIC
SELECTED
S NOV 20 1981 D
D

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER T-449	2. GOVT ACCESSION NO. AD-A107 669	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A CLOSED FORM LOCAL SOLUTION OF A NONLINEAR STRUCTURAL DESIGN PROBLEM IN TERMS OF THE DESIGN PARAMETERS		5. TYPE OF REPORT & PERIOD COVERED SCIENTIFIC
7. AUTHOR(s) ANTHONY V. FIACCO ABOLFAZL GHAEMI	6. PERFORMING ORG. REPORT NUMBER T-449	
9. PERFORMING ORGANIZATION NAME AND ADDRESS THE GEORGE WASHINGTON UNIVERSITY PROGRAM IN LOGISTICS WASHINGTON, DC 20052	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS	
11. CONTROLLING OFFICE NAME AND ADDRESS OFFICE OF NAVAL RESEARCH CODE 434 ARLINGTON, VA 22217	12. REPORT DATE 10 AUGUST 1981	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	13. NUMBER OF PAGES 22	
	15. SECURITY CLASS. (of this report) UNCLASSIFIED	
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE	
16. DISTRIBUTION STATEMENT (of this Report) APPROVED FOR PUBLIC SALE AND RELEASE; DISTRIBUTION UNLIMITED.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) NONLINEAR PROGRAMMING STRUCTURAL DESIGN PARAMETERS CLOSED FORM SOLUTION		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Using the results of our previous analysis of the corrugated bulkhead model, a closed form of the optimal solution of this model is easily derived as a function of the many design parameters. The analytic solution is valid over large changes in the parameters. This demonstrates the possibility of making very accurate (in this instance, exact) local estimates of a markedly nonlinear parametric solution, once one solution is found,		

(continued)

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

20. ABSTRACT (cont'd)

even though the original problem statement might appear to make this prohibitive. It is shown that the local parametric solution obtained can be extended to include large changes in the parameters, particularly when the parametric perturbations are restricted and highly structured, as is typical in practical applications. Aside from the practical value of such a result, it is reported because this problem is well publicized and has been utilized for some time as a test problem. Knowledge of the precise solution for a range of parameter values provides ideal information for further studies.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Institute for Management Science and Engineering
Program in Logistics

Abstract
of
Serial T-449
10 August 1981

A CLOSED FORM LOCAL SOLUTION OF A
NONLINEAR STRUCTURAL DESIGN PROBLEM IN
TERMS OF THE DESIGN PARAMETERS

by

Anthony V. Fiacco
Abolfazl Ghaemi

Using the results of our previous analysis of the corrugated bulkhead model, a closed form of the optimal solution of this model is easily derived as a function of the many design parameters. The analytic solution is valid over large changes in the parameters. This demonstrates the possibility of making very accurate (in this instance, exact) local estimates of a markedly nonlinear parametric solution, once one solution is found, even though the original problem statement might appear to make this prohibitive. It is shown that the local parametric solution obtained can be extended to include large changes in the parameters, particularly when the parametric perturbations are restricted and highly structured, as is typical in practical applications. Aside from the practical value of such a result, it is reported because this problem is well publicized and has been utilized for some time as a test problem. Knowledge of the precise solution for a range of parameter values provides ideal information for further studies.

Research Supported by
Contract N00014-75-C-0729
Project NR 347 020
Office of Naval Research

TABLE OF CONTENTS

Abstract	1
1. THE PROBLEM	1
2. DERIVATION OF CLOSED FORM SOLUTION	11
3. NUMERICAL CORROBORATION	14
4. DEVELOPMENT OF A COMPLETE PARAMETER SOLUTION	16
REFERENCES	19

THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Institute for Management Science and Engineering

A CLOSED FORM LOCAL SOLUTION OF A
NONLINEAR STRUCTURAL DESIGN PROBLEM IN
TERMS OF THE DESIGN PARAMETERS

by

Anthony V. Fiacco
Abolfazl Ghaemi

1. THE PROBLEM

The purpose of this study is to further characterize the optimal solution behavior of a nonlinear programming model for the optimal sizing of a vertically corrugated transverse bulkhead of an oil tanker, presented in [1,Ch. 6] and subsequently analyzed by the authors [2]. The objective of the model is to determine the dimensions of the bulkhead such that certain engineering constraints are met, with minimum possible bulkhead weight.

Figures 1 and 2 depict the configuration of the bulkhead and design variables, respectively. Figure 3, together with Table 1, defines the model parameters and their values.

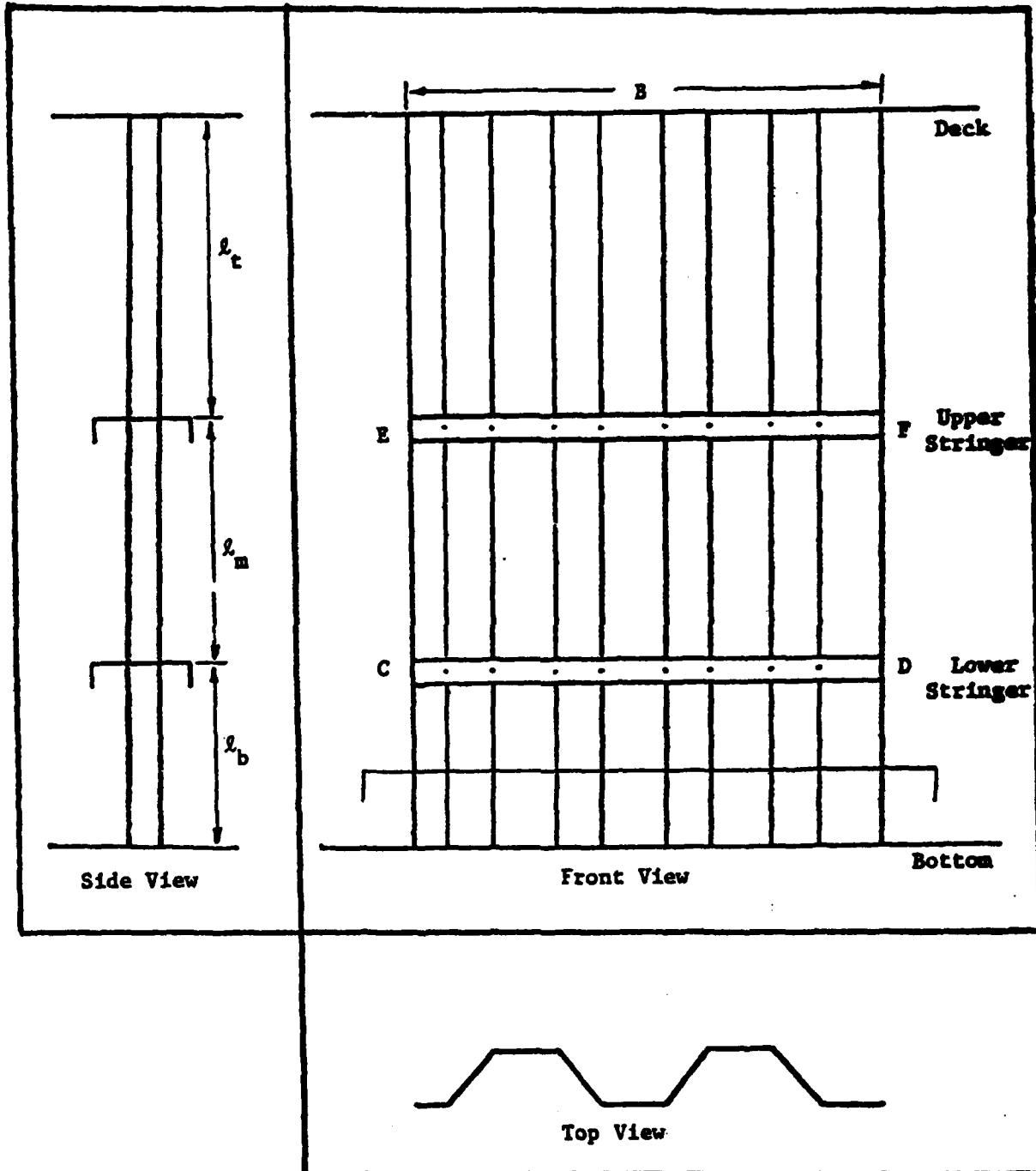


Figure 1. Vertical corrugated transverse bulkhead.

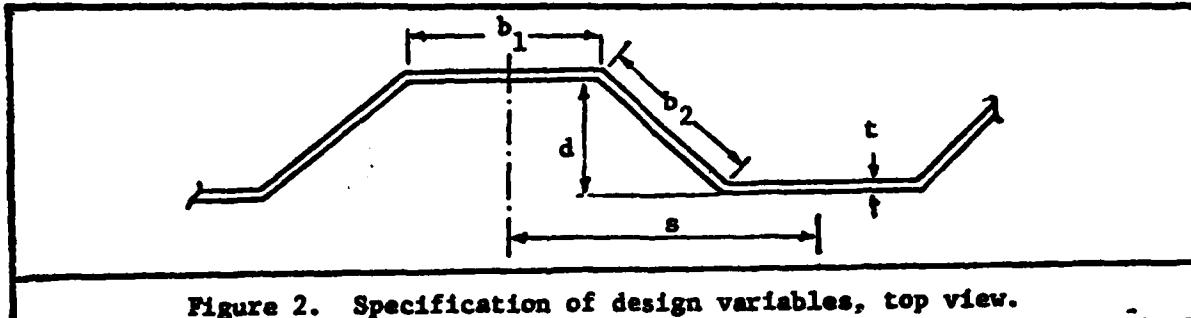


Figure 2. Specification of design variables, top view.

$x_1 = b_1 = \text{width of flange}$

$x_2 = b_2 = \text{length of web}$

$x_3 = d = \text{depth of corrugation}$

$x_4 = t_t = \text{thickness of top panel}$

$x_5 = t_m = \text{thickness of middle panel}$

$x_6 = t_b = \text{thickness of bottom panel.}$

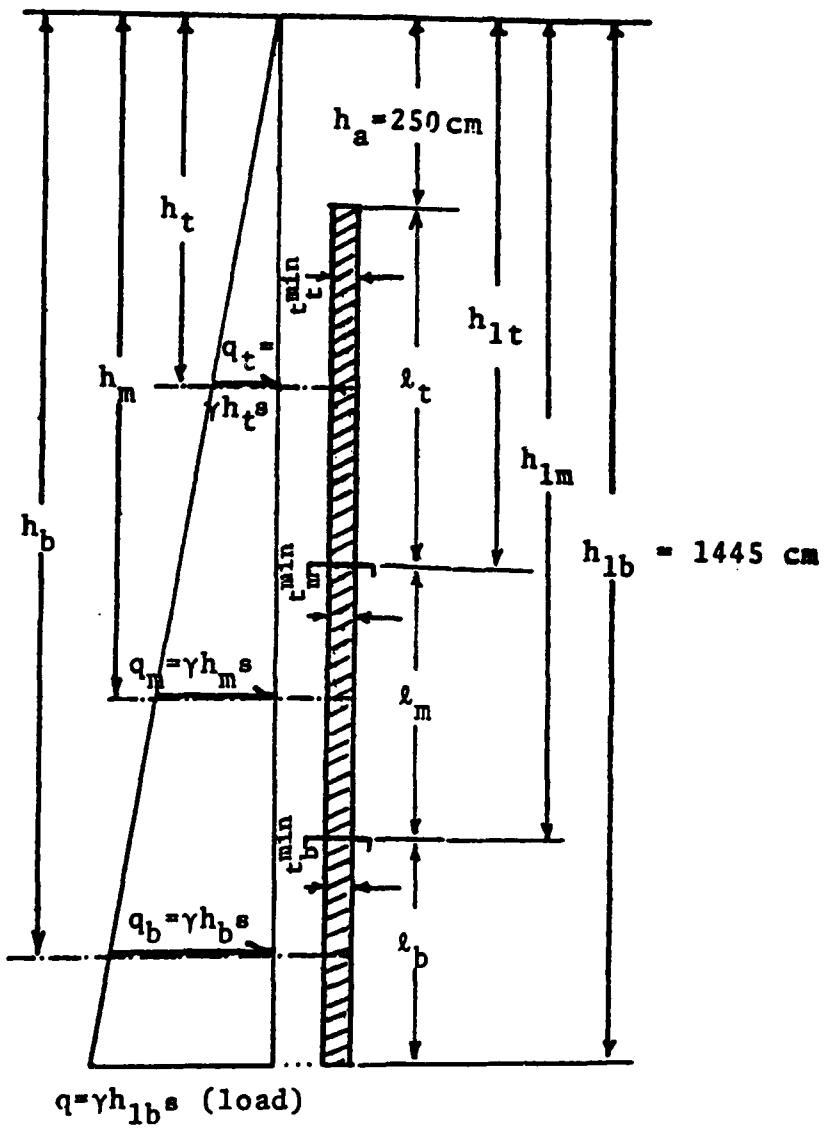


Figure 3. Specification of some of the design parameters and indication of load levels, side view.

TABLE 1
PROBLEM DESIGN PARAMETERS
(MODEL INPUT DATA)

No.	Parameter	Meaning	Value
1	Γ	Weight per unit volume of the material	7.85 grm/cm ³
2	B	Width of the panel	476 cm
3	l_t	Length of the top panel	495 cm
4	l_m	Length of the middle panel	385 cm
5	l_b	Length of the bottom panel	315 cm
6	h_a	Distance between free liquid level and top of structure	250 cm
7	h_t	Distance between free liquid level and middle of top panel	498 cm
8	h_m	Distance between free liquid level and middle of middle panel	938 cm
9	h_b	Distance between free liquid level and middle of bottom panel	1288 cm
10	h_{lt}	Distance between free liquid level and base of top panel	745 cm
11	h_{lm}	Distance between free liquid level and base of middle panel	1130 cm
12	h_{lb}	Distance between free liquid level and base of bottom panel	1445 cm
13	t_t^{\min}	Minimum allowable thickness of top panel	1.05 cm
14	t_m^{\min}	Minimum allowable thickness of middle panel	1.05 cm

Table 1 -- Continued

No.	Parameter	Meaning	Value
15	t_b^{\min}	Minimum allowable thickness of bottom panel	1.05 cm
16	e	Effectiveness of the flange (dimensionless)	0.8
17	k_1	A derived coefficient (function of maximum allowable bending stress)	$6.94 \times 10^{-8} \text{ cm}^{-1}$
18	k_2	Corrosion coefficient	0.15 cm

The resulting nonlinear programming problem obtained in [1] takes the following form.

$$\min_x f \equiv PB(x_1 + x_2)(\ell_t x_4 + \ell_m x_5 + \ell_b x_6) [x_1 + (x_2^2 - x_3^2)^{1/2}]^{-1} \text{ (weight)}$$

subject to

(i) Geometrical constraint:

$$g_1 \equiv x_2 - x_3 \geq 0 ;$$

(ii) Bending stress constraints:

$$g_2 \equiv x_2 x_3 x_4 + 3e x_1 x_3 x_4 - 6k_1 h_t l_t^2 [x_1 + (x_2^2 - x_3^2)^{1/2}] \geq 0 ;$$

$$g_3 \equiv x_2 x_3 x_5 + 3e x_1 x_3 x_5 - 6k_1 h_m l_m^2 [x_1 + (x_2^2 - x_3^2)^{1/2}] \geq 0 ;$$

$$g_4 \equiv x_2 x_3 x_6 + 3e x_1 x_3 x_6 - 6k_1 h_b l_b^2 [x_1 + (x_2^2 - x_3^2)^{1/2}] \geq 0 ;$$

(iii) Moment of inertia constraints:

$$g_5 \equiv x_2 x_3^2 x_4 + 3e x_1 x_3^2 x_4 - 26.4(k_1 h_t l_t^2)^{4/3} [x_1 + (x_2^2 - x_3^2)^{1/2}]^{4/3} \geq 0 ;$$

$$g_6 \equiv x_2 x_3^2 x_5 + 3e x_1 x_3^2 x_5 - 26.4(k_1 h_m l_m^2)^{4/3} [x_1 + (x_2^2 - x_3^2)^{1/2}]^{4/3} \geq 0 ;$$

$$g_7 \equiv x_2 x_3^2 x_6 + 3e x_1 x_3^2 x_6 - 26.4(k_1 h_b l_b^2)^{4/3} [x_1 + (x_2^2 - x_3^2)^{1/2}]^{4/3} \geq 0 ;$$

(iv) Thickness requirement constraints:

$$g_8 \equiv x_4 - t_t^{\min} \geq 0 ;$$

$$g_9 \equiv x_4 - [.39 + 1.05(.01h_{1t})^{1/2}(.01x_1) + k_2] \geq 0 ;$$

$$g_{10} \equiv x_4 - [.39 + 1.05(.01h_{1t})^{1/2}(.01x_2) + k_2] \geq 0 ;$$

$$g_{11} \equiv x_5 - t_m^{\min} \geq 0 ;$$

$$g_{12} \equiv x_5 - [.39 + 1.05(.01h_{1m})^{1/2}(.01x_1) + k_2] \geq 0 ;$$

$$g_{13} \equiv x_5 - [.39 + 1.05(.01h_{1m})^{1/2}(.01x_2) + k_2] \geq 0 ;$$

$$g_{14} \equiv x_6 - t_b^{\min} \geq 0 ;$$

$$g_{15} \equiv x_6 - [.39 + 1.05(.01h_{1b})^{1/2}(.01x_1) + k_2] \geq 0 ;$$

$$g_{16} \equiv x_6 - [.39 + 1.05(.01h_{1b})^{1/2}(.01x_2) + k_2] \geq 0 ;$$

(v) Natural constraints:

$$g_{17}, g_{18}: x_i \geq 0 \quad i = 1, 3$$

Table 2 gives the optimal solution and Table 3 shows the constraint values and Lagrange multipliers corresponding to this solution. These results were obtained in our previous study [1].

TABLE 2

OPTIMAL SOLUTIONS OF THE MODEL
WITH PARAMETER VALUES GIVEN IN TABLE 1

$$x_1 = b_1 = 57.82 \text{ cm}$$

$$x_2 = b_2 = 57.82 "$$

$$x_3 = d = 35.69 "$$

$$x_4 = t_t = 1.05 "$$

$$x_5 = t_m = 1.05 "$$

$$x_6 = t_b = 1.05 "$$

$$f = w = 5.25 \text{ tons}$$

TABLE 3
LAGRANGE MULTIPLIERS AND CONSTRAINT VALUES AT THE OPTIMAL SOLUTION
(excluding nonnegativity constraints)

Constraint s_i	Value of Constraint s_i	Value of Lagrange Multiplier u_i
s_1	22.13	$.4518784 \times 10^{-2}$
s_2	2117.22	$.4723479 \times 10^{-4}$
s_3	1385.41	$.7218857 \times 10^{-4}$
s_4	1868.92	$.5351946 \times 10^{-4}$
s_5	41993.15	$.2381633 \times 10^{-5}$
t_{s_6}	.043	$.2300013 \times 10^1$
s_7	27945.57	$.3580375 \times 10^{-5}$
t_{s_8}	$.48 \times 10^{-5}$	$.2078305 \times 10^7$
s_9	.25	.3940568
s_{10}	.25	.3940570
$t_{s_{11}}$	$.96 \times 10^{-5}$	$.1040634 \times 10^7$
s_{12}	.10	.9604345
s_{13}	.10	.9604360
$t_{s_{14}}$	$.49 \times 10^{-4}$	$.2029215 \times 10^6$
$t_{s_{15}}$	$.31 \times 10^{-4}$	$.3221582 \times 10^6$
$t_{s_{16}}$	$.13 \times 10^{-4}$	$.7974950 \times 10^6$

*Binding constraints.

As indicated in Table 3, constraints g_6 , g_8 , g_{11} and $g_{14} - g_{16}$ are binding. It was shown in [2] that the corresponding gradients of these functions are linearly independent. The optimal solution of the model, with the given data base (Table 1), thus corresponds to the locally unique solution of the following nondegenerate system of nonlinear equations:

$$\begin{aligned}
 g_6 &= x_2^2 x_3^2 x_5 + 3e x_1 x_3^2 x_5 - 26.4(k_1 h_m l_m)^{2/3} [x_1 + (x_2^2 - x_3^2)^{1/2}]^{4/3} = 0 \\
 g_8 &= x_4 - t_t^{\min} = 0 \\
 g_{11} &= x_5 - t_m^{\min} = 0 \\
 g_{14} &= x_6 - t_b^{\min} = 0 \\
 g_{15} &= x_6 - [(0.39)(1.05)(0.01h_{lb})^{1/2}(0.01x_1) + k_2] = 0 \\
 g_{16} &= x_6 - [(0.39)(1.05)(0.01h_{lb})^{1/2}(0.01x_2) + k_2] = 0
 \end{aligned} \tag{1}$$

In [2] we determined that, with parameter perturbations as large as two percent of the parameter base values, the set of binding constraints remains unchanged. This means that, for relatively large changes of data in a neighborhood of the data base given in Table 1, the optimal parametric solution of the nonlinear program is unique and must satisfy the system of equation (1).

In the next section, we obtain a closed form solution for this system of nonlinear equations.

2. DERIVATION OF CLOSED FORM SOLUTION

For notational simplicity, we denote the coefficients of Equations

(1) as follows:

$$c_1 = 3e$$

$$c_2 = 26.4(k_1 h_m l_m^2)^{4/3}$$

$$c_3 = k_2$$

$$c_4 = t_t^{\min}$$

$$c_5 = t_m^{\min}$$

$$c_6 = t_b^{\min}$$

$$c_7 = (.39)(1.05)(.01h_{1b})^{1/2}(.01)$$

Thus, the system of equation (1) becomes

$$g_6 : x_2 x_3^2 x_5 + c_1 x_1 x_3^2 x_5 - c_2 [x_1 + (x_2^2 - x_3^2)^{1/2}]^{4/3} = 0$$

$$g_8 : x_4 - c_4 = 0$$

$$g_{11} : x_5 - c_5 = 0$$

$$g_{14} : x_6 - c_6 = 0$$

$$g_{15} : x_6 - c_7 x_1 - c_3 = 0$$

$$g_{16} : x_6 - c_7 x_2 - c_3 = 0$$

From g_8 , g_{11} and g_{14} we obtain

$$x_4 = c_4, x_5 = c_5 \text{ and } x_6 = c_6 \quad (3)$$

From g_{15} and g_{16} and (3),

$$x_1 = x_2 = (c_6 - c_3)/c_7 = c_8 \quad (4)$$

Substituting x_1 and x_2 from (4) and x_5 from (3) in g_6 , we obtain

$$(c_5 c_8 + c_1 c_8 c_5) x_3^2 - c_2 [c_8 + (c_8^2 - x_3^2)^{1/2}]^{4/3} = 0$$

or

$$(c_5 c_8 (1 + c_1)/c_2)^{3/4} x_3^{3/2} = [c_8 + (c_8^2 - x_3^2)^{1/2}] .$$

$$\text{Letting } c_9 = \left(\frac{c_5 c_8 (1 + c_1)}{c_2} \right)^{3/4}, \quad (5)$$

then subtracting c_8 from both sides of the equation and squaring,
yields

$$c_9^2 x_3^3 + x_3^2 - 2c_8 c_9 x_3^{3/2} = 0$$

or

$$x_3^{3/2} [c_9^2 x_3^{3/2} + x_3^{1/2} - 2c_8 c_9] = 0$$

Since we must have $x_3 > 0$ near the given base value solution and
since we may assume $c_1, c_5, c_2 > 0$ and $c_6 > c_3$, then $c_8 > 0$

and $c_9 \neq 0$, so

$$(\sqrt{x_3})^3 + \frac{1}{c_9^2} \sqrt{x_3} - \frac{2c_8}{c_9} = 0 \quad (6)$$

a reduced cubic equation in $\sqrt{x_3}$.

According to a classical result discovered by Ferro and first published by Cardan [3], a reduced cubic equation of the form

$$y^3 + ay + b = 0 \quad (7)$$

with $(\frac{b}{2})^2 + (\frac{a}{3})^3 > 0$ has only one real root.

Since in (6), $a = -\frac{1}{c_9^2}$ is always positive, $(\frac{b}{2})^2 + (\frac{a}{3})^3$ is

always positive for our problem for all changes of these coefficients that result from any (feasible) model parameter perturbations.

The unique real root of (7) is given by Cardan's formula as follows:

$$y = \left[\frac{-b}{2} + \sqrt{\left(\frac{b}{2}\right)^2 + \left(\frac{a}{3}\right)^3} \right]^{1/3} + \left[\frac{-b}{2} - \sqrt{\left(\frac{b}{2}\right)^2 + \left(\frac{a}{3}\right)^3} \right]^{1/3}$$

thus, the solution for x_3 from (6) is

$$x_3 = \left\{ \left[\frac{c_8}{c_9} + \left(\frac{c_8^2}{c_9^2} + \frac{1}{27c_9^6} \right)^{1/2} \right]^{1/3} \right. \\ \left. + \left[\frac{c_8}{c_9} - \left(\frac{c_8^2}{c_9^2} + \frac{1}{27c_9^6} \right)^{1/2} \right]^{1/3} \right\}^2 \quad (8)$$

We have thus obtained a closed form parametric solution for the optimal problem variables near the given initial parameter values. Substituting $x_1 - x_6$ from (3), (4) and (8) in the objective function f of the model well of course, yield the closed form formula for the optimal value function.

3. NUMERICAL CORROBORATION

The closed formulas we have derived provide an opportunity to check the computer solution. In this section, we use the data base given in Table 1 and the solution formulas we have derived to calculate the optimal solution components and objective function value, and compare these with those obtained by the computer solution and listed in Table 2.

Using the data in Table 1, we obtain the following values for $c_1 - c_7$:

$$c_1 = 2.4, c_2 = 542.311667, c_3 = .15, c_4 = c_5 = c_6 = 1.05$$

and $c_7 = .015566$.

$$\text{From (4), } c_8 = \frac{c_6 - c_3}{c_7} = 57.818322,$$

and
from (5), $c_9 = \left(\frac{c_5 c_8 (1+c_1)}{c_2} \right)^{3/4} = .484578$.

Substitution of c_8 and c_9 in (8) yields.

$$x_3 = (5.973894)^2 = 35.687409.$$

Thus, the analytically calculated solution vector is, to two decimal points of accuracy,

$$x = (57.82, 57.82, 35.69, 1.05, 1.05, 1.05),$$

which corresponds precisely to the computer solution given in Table 2. Substantiation of these values of x in the objective function f and rounding to two decimal points yields a value of 5.25, again in precise correspondence with the computer solution.

As an example of a verification of previously derived sensitivity results, we calculate the sensitivity of the optimal value function to the parameter t_{\min}^t , the right-hand side of the constraint g_8 which we denoted by c_4 .

Denoting x_3 by c_{10} and substituting the closed form solutions for x_i $i=1,6$ in the objective function f , we obtain the formula for the optimal value function,

$$f^*(\cdot) = 2\Gamma B c_8 (\lambda_t c_4 + \lambda_u c_5 + \lambda_b c_6) / \sqrt{(c_8^2 - c_{10}^2)^{1/2} + c_8} . \quad (9)$$

Thus,

$$\frac{\partial f^*(\cdot)}{\partial c_4} = 2\Gamma B c_8 \lambda_t \left[(c_8^2 - c_{10}^2)^{1/2} + c_8 \right]^{1/2} .$$

Substituting in (9) the numerical values of Γ , B and λ_t given in Table 1 and the calculated values of c_8 and c_{10} given in the above formulas, we obtain

$$\frac{\partial f^*(\cdot)}{\partial c_4} = 2070337.126 .$$

The computer solution obtained in [1] was 2078305. This corresponds with the calculated Lagrange multipliers value associated with constraint g_8 , which is given in Table 3. These calculations are in close agreement, in terms of relative error.

4. DEVELOPMENT OF A COMPLETE PARAMETRIC SOLUTION

A careful study of the local solution that has been obtained and the constraint structure indicates that a great deal of progress can be made towards obtaining a global parametric solution, particularly if the parameter perturbations are restricted to those that typically arise in practice. For example, perturbations of considerable interest are based on the assumption that some or all of the parameters change "proportionately." e.g., if a given parameter vector is denoted by p , then a perturbed value of p would be given by $p + \alpha p = (1 + \alpha)p = \beta p$, where α is the proportionality factor and $\beta = 1 + \alpha$, a scalar. With this restriction, all the parameters that change are functions of the single scalar valued parameter β and the perturbation analysis is enormously simplified. In the present example, given this type of perturbation, we could readily track the course of the closed form parametric solution that we have obtained as a function of the single parameter β , as β increases or decreases. As long as the given binding constraints remain binding, the given system of equations hold and the given parametric solution would apply. When a new constraint becomes binding at a new value of β , we would have to introduce a new equation and when a binding constraint becomes nonbinding, the corresponding equation would be removed from the binding constraint equations. Given the relative simplicity of our closed form solution and the exploitable structure of the constraints, as we shall indicate, a complete closed form parametric solution could probably be developed over the domain of parameter values of interest.

At a general level, note that constraints g_1 and $g_8 - g_{16}$ are simple linear constraints. Each of these constraints that are binding will either determine the value of a variable or determine one variable in terms of another, thus, each removing one degree of freedom in the solution. If any of the constraints $g_2 - g_4$ are binding, they can each be solved for any variable in terms of the others involved in the particular constraint. Finally, if either of $g_5 - g_7$ are binding

then, x_1 , x_4 , x_5 , or x_6 , can be expressed in terms of the other variables, and if $x_1 = x_2$, then the remaining variables, x_2 and x_3 , as we have shown, can be solved in terms of the other variables. Thus, the preponderance of the various possible combinations of equations that can arise from the constraints that are binding at a solution corresponding to a given parameter value can be resolved in closed form.

We conclude with an extension of the local closed solution that we have obtained to illustrate our observations. Consider changing the parameter t_t^{\min} from its base value 1.05 cm, leaving the other parameters fixed as given in Table 1. Denote the parameter vector by ϵ , so that ϵ_i denotes the ith parameter value, and the closed form solution vector that we have obtained by $x(\epsilon)$. If $\epsilon_{13} = t_t^{\min}$ increases from its base value to any larger value \bar{t}_t^{\min} , with the other ϵ_i fixed at their initial values ϵ_i^* , then the optimal parametric solution vector is given by $x_4(\epsilon) = \bar{t}_t^{\min}$, with the other components $x_i(\epsilon)$ of the solution unchanged. On the other hand, if t_t^{\min} decreases to \underline{t}_t^{\min} , with the other parameter values fixed at their given initial values, then we find that the optimal parametric solution is given by $x_4(\epsilon) = \underline{t}_t^{\min}$, with no change in all the other components, providing $\underline{t}_t^{\min} - t_t^{\min}$ is small. However, the nonbinding constraints g_2 , g_5 , g_9 and g_{10} evaluated at this solution decrease so the solution remains optimal providing

$$t_t^{\min} \geq \max(m_2, m_5, m_9, m_{10})$$

where m_i is the quantity involved in the ith constraint when the ith constraint is equivalently expressed in the form $x_4 \geq m_i[x(\epsilon), \epsilon]$, $i = 2, 5, 9, 10$, and the m_i are evaluated at the given solution $x(\epsilon)$

for the given parameter data base $\bar{\epsilon}$. Hence, the given closed form parametric solution is valid for $t_t^{\min} \geq \underline{t}_t^{\min}$, where

$$\underline{t}_t^{\min} = \max(m_2, m_5, m_9, m_{10}) \quad (10)$$

Clearly, $g_6, g_8, g_{11}, g_{14}, g_{15}$ and g_{16} will remain binding, while g_2, g_5, g_9 or g_{10} will become binding when $x_4 = \underline{t}_t^{\min}$.

We do not pursue other possible extensions of the parametric solution. A moral of the foregoing analysis is that closed form parametric solutions are not the exclusive province of "toy" problems. Closed form solutions provide precise and complete information, a goal that may be perfunctorily relinquished for "practical problems." Even if a complete parametric solution cannot be obtained, various components may be possible to derive in closed form over parameter perturbations of interest, thus greatly simplifying further analysis. In this problem, for example, even if $x_3(\epsilon)$ could not have been obtained, the other components of the local parametric solution are immediately available in closed form.

Obviously, the existence of a complete closed form parametric solution is a fortunate happenstance. Components of nonlinear programming parametric solutions will inevitably have to be estimated, perhaps most aptly by predictor-corrector techniques and their contemporary refinements, e.g., continuation methods [4], [5]. Nonetheless, experience suggests that some closed form functional relationships characterizing a solution will inevitably be available and are worth exploiting once a solution has been determined.

REFERENCES

- [1] BRACKEN, J. and G. P. MCCORMICK, (1968). Selected Applications of Nonlinear Programming, John Wiley.
- [2] FIACCO, A. V. and A. GHAEMI, (1980). Sensitivity analysis of a nonlinear structural design problem, Technical Paper Serial T-413, Institute for Management Science and Engineering, The George Washington University.
- [3] JAMES, GLENN and ROBERT C. JAMES, (1959). Mathematics Dictionary. D. Van Nostrand Company, Inc., pp. 40-41.
- [4] ORTEGA, J. M., and W. C. RHEINBOLDT, (1970). Iterative Solution of Nonlinear Equations in Several Variables. Academic Press.
- [5] WACKER, H. ed. (1978). Continuation Methods. Proceedings of a symposium at the University of Linz, Austria, October 3-4, 1977. Academic Press.

THE GEORGE WASHINGTON UNIVERSITY
Program in Logistics
Distribution List for Technical Papers

The George Washington University Office of Sponsored Research Gelman Library Vice President H. F. Bright Dean Harold Liebowitz Dean Henry Solomon	Armed Forces Industrial College	Case Western Reserve University Prof. B. T. Dean Prof. M. Mesarovich
ONR Chief of Naval Research (Codes 200, 434) Resident Representative	Armed Forces Staff College	Cornell University Prof. S. L. Bechteler Prof. R. W. Conway Prof. Andrew Schultz, Jr.
OPNAV OP-40 DCNO, Logistics Navy Dept Library NAVDATA Automation Cmd	Army War College Library Carlisle Barracks	Cowles Foundation for Research in Economics Prof. Martin Shubik
Naval Aviation Integrated Log Support	Army Cmd & Gen Staff College	Florida State University Prof. R. A. Bradley
NARDAC Tech Library	Army Logistics Mgt Center Fort Lee	Harvard University Prof. W. G. Cochran Prof. Arthur Schleifer, Jr.
Naval Electronics Lab Library	Commanding Officer, USALDSRA New Cumberland Army Depot	Princeton University Prof. A. W. Tucker Prof. J. W. Tukey Prof. Geoffrey S. Watson
Naval Facilities Eng Cmd Tech Library	Army Inventory Res Ofc Philadelphia	Purdue University Prof. S. S. Gupta Prof. H. Rubin Prof. Andrew Whinston
Naval Ordnance Station Louisville, Ky. Indian Head, Md.	Army Trans Material Cmd TCMAC-ASDT	Stanford University Prof. T. W. Anderson Prof. Kenneth Arrow Prof. G. B. Dantzig Prof. F. S. Hillier Prof. D. L. Iglehart Prof. Samuel Karlin Prof. G. J. Lieberman Prof. Herbert Solomon Prof. A. F. Veinott, Jr.
Naval Ordnance Sys Cmd Library	Air Force Headquarters AFADS-3 LEXY SAF/ALG	University of California, Berkeley Prof. R. E. Barlow Prof. D. Gale Prof. Jack Kiefer
Naval Research Branch Office Boston Chicago New York Pasadena San Francisco	Griffiss Air Force Base Reliability Analysis Center	University of California, Los Angeles Prof. R. R. O'Neill
Naval Ship Eng Center Philadelphia, Pa.	Gunter Air Force Base AFLMC/XR	University of North Carolina Prof. W. L. Smith Prof. M. R. Leadbetter
Naval Ship Res & Dev Center	Maxwell Air Force Base Library	University of Pennsylvania Prof. Russell Ackoff
Naval Sea Systems Command PMS 30611 Tech Library Code 073	Wright-Patterson Air Force Base AFLC/OA Research Sch Log AFALD/XR	University of Texas Institute for Computing Science and Computer Applications
Naval Supply Systems Command Library Operations and Inventory Analysis	Defense Technical Info Center	Yale University Prof. F. T. Ansecombe Prof. H. Scott
Naval War College Library Newport	National Academy of Sciences Maritime Transportation Res Bd Lib	Prof. T. W. Birnbaum University of Washington
BUPERS Tech Library	National Bureau of Standards Dr. B. H. Colvin Dr. Joan Rosenblatt	Prof. B. H. Bissinger The Pennsylvania State University
FMSO	National Science Foundation	Prof. Seth Bonder University of Michigan
USN Ammo Depot Earle	National Security Agency	Prof. G. E. Box University of Wisconsin
USN Postgrad School Monterey Library Dr. Jack R. Borsting Prof. C. R. Jones	Weapons Systems Evaluation Group	Dr. Jerome Bracken Institute for Defense Analyses
US Coast Guard Academy Capt Jimmie D. Woods	British Navy Staff	
US Marine Corps Commandant Deputy Chief of Staff, R&D	National Defense Hdqtrs, Ottawa Logistics, OR Analysis Estab	
Marine Corps School Quantico Landing Force Dev Ctr Logistics Officer	American Power Jet Co George Chernowitz	
	General Dynamics, Pomona	
	General Research Corp Library	
	Logistics Management Institute Dr. Murray A. Geisler	
	Rand Corporation Library Mr. William P. Hutzler	
	Carnegie-Mellon University Dean H. A. Simon Prof. G. Thompson	

Continued

Prof A. Charnes
University of Texas

Prof H. Chernoff
Mass Institute of Technology

Prof Arthur Cohen
Rutgers - The State University

Mr Wallace M. Cohen
US General Accounting Office

Prof C. Derman
Columbia University

Prof Masao Fukushima
Kyoto University

Prof Saul I. Gass
University of Maryland

Dr Donald P. Gaver
Carmel, California

Prof Amrit L. Goel
Syracuse University

Prof J. F. Hannan
Michigan State University

Prof H. O. Hartley
Texas A & M Foundation

Prof W. M. Hirsch
Courant Institute

Dr Alan J. Hoffman
IBM, Yorktown Heights

Prof John R. Isbell
SUNY, Amherst

Dr J. L. Jain
University of Delhi

Prof J. H. K. Kao
Polytech Institute of New York

Prof W. Kruskal
University of Chicago

Mr S. Kumar
University of Madras

Prof C. L. Lemke
Rensselaer Polytech Institute

Prof Loynes
University of Sheffield, England

Prof Tom Maul
Kowloon, Hong Kong

Prof Steven Nahmias
University of Santa Clara

Prof D. B. Owen
Southern Methodist University

Prof P. R. Parathasarathy
Indian Institute of Technology

Prof E. Parzen
Texas A & M University

Prof H. O. Posten
University of Connecticut

Prof R. Remage, Jr.
University of Delaware

Prof Hans Riedwyl
University of Berne

Mr David Rosenblatt
Washington, D. C.

Prof M. Rosenblatt
University of California, San Diego

Prof Alan J. Rowe
University of Southern California

Prof A. H. Rubenstein
Northwestern University

Prof Thomas L. Saaty
University of Pittsburgh

Dr M. E. Salveson
West Los Angeles

Prof Gary Sodder
University of Minnesota

Prof Edward A. Silver
University of Waterloo, Canada

Prof Rosedith Sitpreaves
Washington, DC

LTG G. L. Slyman, MSC
Department of the Army

Prof M. J. Sobel
Georgia Inst of Technology

Prof R. M. Thrall
Rice University

Dr S. Vajda
University of Sussex, England

Prof T. M. Whitin
Wesleyan University

Prof Jacob Wolfowitz
University of South Florida

Prof Max A. Woodbury
Duke University

Prof S. Zacks
SUNY, Binghamton

Dr Israel Zang
Tel-Aviv University

February 1981

**DATE
ILME**